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AEROSPACE SYSTEMS AND MISSION ANALYSIS RESEARCH

Status Report for the Period

1 February through 30 September 1965

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I. INTRODUCTION

Research has continued during this period primarily on methods for optimization of space flight trajectories pertinent to the analysis of the unmanned Earth-Mars round trip mission. Some work has continued on specific propulsion system studies.

Personnel associated with the work remain the same as discussed in the previous status report. Dr. Paul M. Lion, a Sloan Post-doctoral Fellow in this Department for 1965-66, has indicated a willingness to become involved in our Program in the area of space flight trajectory analysis. Mr. Ernst Dickmanns, a NASA International Fellow, worked with us during the past summer before his return to Germany.

A number of consultants have been added to the Program. Professor Arthur E. Bryson of Harvard who is to be Hunsaker Professor of Aeronautical Engineering at Massachusetts Institute of Technology for Academic Year 1965-66 and Professor George Leitmann of the University of California, Berkeley will consult in their well-known specialty of optimal control theory from time to time. Analytic Mechanics Associates, Inc., including Mr. Samuel Pines, Dr. Henry Kelley, Dr. Henry Wolf, Dr. Walter Denham and others, has been engaged for assistance in development of applied mathematics and computer programming.

The work itself is presented in the following sections.

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II. SPACEFLIGHT TRAJECTORY ANALYSIS

We are now fully involved in dealing with the many complexities required in developing the mathematical approaches and computer programs for the optimization of analysis of spaceflight trajectories. The focus is still the unmanned Earth-Mars round trip mission. We continue to deal with the constant thrust and jet velocity characteristic of the propulsion systems over a range of parameters for chemical rockets through nuclear rockets to electric thrusters which complicates the mathematics and the programs. We have also introduced the actual planetary elements and ephemerides which involves three-dimensional orbital configurations. Work is also underway on planetary escape and capture trajectories, including patching on to optimized heliocentric transfer trajectories.

A. Accomplishments to Date

1. The previous status report (1 February 1964 - 31 January 1965) contained a preliminary discussion of an investigation directed at removing a prime difficulty in current applications of the classical, indirect, calculus-of-variations method to trajectory optimization. This difficulty results from the fact that numerical solution of the resultant two-point boundary value problem requires knowledge of the initial (or final) values of certain of the Lagrangian multiplier functions $\lambda_i(t)$ for numerical integration of the differential equations. In practice, except for a few simple cases of limited practical interest, complete ignorance exists concerning these values. As a consequence, the current state-of-the-art in application of the classical method is based upon guessing or otherwise estimating a set of initial values $\lambda_i(0)$, for forward integration, for example, and reliance upon an iterative computational scheme to converge

these initial guesses to the correct but unknown values so as to satisfy the given boundary conditions. Unfortunately, a high sensitivity of terminal values to the $\lambda_i(0)$ is characteristic, which makes convergence very difficult at best, and often not attainable. For these reasons, this method has been termed "an art" by Kelley (1)* and is considered as having proven largely unsuccessful by Kenneth and McGill (2).

It can now be reported that we have been successful in overcoming this difficulty for the important class of trajectories initially investigated and described below. At present, optimal trajectories for fixed-thrust, fixed exhaust velocity engines, are being calculated without any guessing (or estimation) of the $\lambda_i(0)$. The technique is to use the $\lambda_i(0)$ generated by solution of the optimal impulsive trajectory as the initial $\lambda_i(0)$ for the optimal trajectory with non-impulsive propulsion. Pines pointed out that the impulsive solution should be the limit point of a simply-connected space of initial values $\lambda_i(0)$ for non-impulsive trajectories, as the thrust magnitude approaches infinity, and the thrust duration approaches zero. This conjecture, which appeared to offer a logical and workable approach to the solution of non-impulsive optimal trajectories, stimulated the direction of research carried out by Dr. Handelsman and has been confirmed in theory and by ample numerical results (3).

A paper containing the pertinent theory and numerical results has been written and may be consulted for details (4). A brief summary of

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*Arabic numbers in parentheses refer to the list of References in APPENDIX A.

this paper is as follows: The theory for calculation of the $\lambda_i(t)$ for optimal two-impulse trajectories satisfying the given boundary conditions is given first. From these $\lambda_i(t)$, the transition to non-impulsive optimal trajectories with specified finite-thrust and fixed exhaust velocity propulsion is then derived and demonstrated, using either (1) a sweep in F/m_0 (thrust/initial mass) and V_j (exhaust velocity), or (2) an iteration directly from the impulsive case. Using method (1), the performance of engines with F/m_0 and V_j spanning the range from chemical through nuclear to electric propulsion, can be calculated in a single sweep. This is most useful for comparative analysis of different propulsion systems for a given mission. A number of representative examples are given of optimal, free-space, central-body field, two-dimensional, interplanetary transfers, between both circular and elliptic orbits, and for both fixed-time and open-time cases. Comparison with published results using the gradient and generalized Newton-Raphson methods show excellent agreement.

At the present writing, the above-described body of theory and numerical techniques has been developed for, and is therefore restricted to the class of two-dimensional trajectories with transfer times and angles such that the optimal power control program is on, off, and on. The impulsive trajectory which is the limit point of such non-impulsive trajectories is a single arc with two terminal impulses and one interior coasting arc between the terminal impulses. This class of optimal trajectories may be roughly characterized as having limiting values for transfer angle and time not greatly in excess of 180° and the Hohmann-transfer time, respectively.*

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*The exact value depends mostly upon F/m_0, and somewhat upon V_j.

When these limits are exceeded, then depending upon the initial and terminal orbits, the optimal trajectory may require provision for one or two terminal coasting arcs, or for interior impulses, or for more than one interior coasting arc, etc. This type of trajectory does not have the simple power control program on, off, and on. Accordingly, extension of the technique is under study to trajectories with transfer angles exceeding 180°, termed "hooks," for which the limiting impulsive trajectory may require more than two terminal impulses, or terminal coast periods, or more than one interior coast period.

2. In the course of developing the theoretical techniques and computational capabilities as described above for calculation of optimal finite-thrust trajectories, programs for calculation of optimal impulsive (two-impulse) trajectories have been successfully completed as by-products. These programs are useful in their own right for mission analysis using impulsive-type propulsion, and are identified below:

(a) Fixed-angle, open-time transfers between given terminals (a terminal point is defined by a position and a velocity vector; hence, the transfer angle is also fixed). The development of the theory by Lee (5) is used, which requires location and test of the roots of an eight-order polynomial, whose variable is the semi-latus rectum of the transfer ellipse. The equations actually used for the program are given in Ref. 6. The program is written in double precision. It is mentioned that for a particular but interesting mission where a double minimum exists in the total impulse velocity change, Lee shows unequal minima where equal minima should be obtained, according to study by Mr. S. M. Rocklin (7). (Also see APPENDIX B for a list of current MARS Memos.)

(b) Fixed-time, open-angle transfers between coplanar circular orbits. This has been solved and programmed by Mr. G. A. Hazelrigg, using a Newton-type iterative solution of Kepler's equation (8). This program is written in double precision.

(c) Fixed-time, fixed-angle transfers between given terminals; i.e., rendezvous missions, between coplanar elliptic orbits. This was solved and programmed by Mr. P. J. Wallack, using a Newton-type iterative solution and appropriate forms of Lambert's equation given by Jordan (9).

3. A number of impulsive and non-impulsive optimal (minimum-propellant, unless otherwise stated) heliocentric trajectories have been calculated for various interplanetary missions (all in two dimensions) of interest, as follows:

(a) Fixed-time, fixed-angle, rendezvous, Earth-to-Mars, elliptic planetary orbits, using two-dimensionalized ephemerides (the orbit of Mars is projected onto that of Earth's). The ephemerides equations developed by Mr. G. A. Hazelrigg (10), and programmed by Mr. P. J. Wallack, furnish required data such as planetary position, angle, velocity, date, time, for computation of optimal trajectories between Earth and Mars for the 1970-1990 time period.

(b) Fixed-time, fixed-angle, Earth-to-Mars, and Mars-to-Earth, circular orbits.

(c) Fixed-time, open-angle, Earth-to-Mars, circular orbits.

(d) Fixed-time, fixed-angle, Mars-to-Earth, circular orbits.

(e) Minimum-time, open-angle, Earth-to-Mars, circular orbits.

(f) Fixed-angle, open time, Earth-to-Venus, Venus-to-Earth, and Earth-to-Mercury, all circular orbits.

(g) Fixed-angle, open-time, Earth-to-orbits with radii varying from 0.2×10^{11} to 0.4×10^{12} meters (approximately 0.13 to 2.7 A.U.), circular orbits.

4. A highly useful generalized program for 35 mm film recording of the IBM 7094 computer cathode-ray tube output has been completed by Mr. P. J. Wallack. This provides for plotting of planetary orbits, impulsive and non-impulsive trajectories, graphs of specified functions of the state and adjoint variables, (λ_i) individually or composite, to name a few of the interesting and useful capabilities.

5. All programs have been converted to Fortran IV.

B. Work in Progress

At the present writing the following work is in progress:

1. Analytic solutions of the optimized flight equations during coast are available, and for improved accuracy, can be used to replace numerical integration of these equations. These solutions have been programmed, "debugged," and are presently being interfaced with the programs for the initial and final power-on phases of the trajectory.

2. Investigation of alternative analytic and computation methods other than the indirect method of the calculus-of-variations and the digital computer, respectively. For example, the generalized Newton-Raphson method (2, 11) is under study as a very promising direct method. This method requires a starting vector which must be guessed, but its power and rapidity of convergence are impressive. It is possible that an appropriate combination of the indirect method developed during this study and a direct method such as the generalized Newton-Raphson may prove to have useful properties. As another example, the application of analog or hybrid (analog-digital) machines

to speed up the computations is also under investigation.

3. Computation of optimal heliocentric transfers proceeds as follows:

(a) Additional trajectories Earth-to-Mars and return, elliptic orbits, two-dimensionalized true ephemerides, in connection with the round-trip mission to Mars under study.

(b) Earth-to-Jupiter, circular orbits, for preliminary study of propulsion requirements for flyby and orbiter missions.

4. Planetocentric escape and capture maneuvers are being studied including patching on to heliocentric transfers using sphere-of-influence concepts and asymptotic matching.

C. Planned Research

1. The iteration scheme used in the computations to date is a fairly standard one (12). The considerable experience gained during the past reporting period in successful calculations of many varied types of optimal trajectories indicates the desirability to develop a faster and more smoothly converging iteration procedure. Improvements in the present scheme and investigation of newer types will be carried out.

2. Extension of the impulsive-iterative method described above (4) to a wider class of trajectories, as follows:

(a) To "hooked" trajectories, which may be encountered in achieving rendezvous in optimized round-trip missions, and also may be required for special one-way missions.

(b) Extension of the present two-dimensional method to three dimensions.

(c) In order to accomplish very fast-time missions, hyperbolic coast arcs become necessary. Extension of the present method, which incorporates only elliptic arcs, to include all possible conics (i.e., ellipses, parabolas and hyperbolas) is planned.

(d) To probe or flyby trajectories. The work to date has been solely on trajectories which match terminal orbital or planetary conditions of position and velocity.

3. Continuation of the planetary escape and capture trajectory analysis work to include optimal patching to heliocentric transfers with three-dimensional and multi-body effects.

III. AEROSPACE SYSTEMS ANALYSIS

During this period we became convinced that it would be necessary to broaden the studies of space propulsion systems so that fundamental and modern methods of systems analysis could be introduced and even extended by application to the propulsion and other vehicle systems involved in the missions of concern of this research. The advanced space propulsion systems, especially those based on nuclear energy, characteristically have more degrees of freedom than the chemical systems and a much greater variety of possible system arrangements. This is especially true of the nuclear power systems and the electric thrustor systems. Just the screening of the higher performing systems from the realm of possibilities and the identification of the significant parameters and their sensitivities will require early abandonment of the old methods involving extensive cross-plotting of results. Systems will need to be described mathematically in all their complexity and programmed for electronic computer analysis. In the meantime, the studies described below are in progress.

A. Nuclear Space Power System Analyses

A joint MSE thesis, Nuclear Space Power System Analyses for the Unmanned Mars Round Trip, has been completed by Lieutenants C. D. Burton and J. A. Evans, who were Navy post-graduate students in this Department during Academic Year 1964-65 (See APPENDIX C List of Publications). This thesis has been placed in a limited distribution category to permit review and further development of its data and conclusions by Mr. Peter Williams of the Nuclear Propulsion Research Laboratory before issuing it as a technical report. The primary intention of this study is the development of parameters for use in the analysis of the Earth-Mars round trip mission.

B. Survey of Electric Space Propulsion Systems

An undergraduate thesis is being completed by Mr. John Slaybaugh, '66, involving a study of the published literature and available technical reports that will survey electric space propulsion systems for their applicability in the 1975-85 period to the unmanned Earth-Mars round trip mission. Work has been devoted during this period to an understanding of the function of the new magnetoplasmadynamic thrustors and to the development of predicted performance parameters for both the thrustor and power conversion systems. Attention will be given also to ion thrustor and power conversion systems.

C. Nuclear Rocket Systems

Studies of the factors influencing the sizing of solid core nuclear rocket systems have been initiated for use in the mission of interest to this research. Much of the literature on nuclear rocket core design is classified so it is planned to make this effort insofar as possible from fundamental principles as found in texts and other published works so that it can be used in some detail in the mission analyses. Primary efforts during this period have been given to a survey of the unclassified literature.

IV. PLANETARY/INTERPLANETARY MISSION ANALYSIS

Although efforts have been made during this period to develop the spaceflight trajectory analysis capability and the propulsion system parameters that are necessary for a first mode analysis of the unmanned Earth-Mars round trip mission, we have not been able to do so. Our insistence on conducting "realistic" mission analyses using actual planetary ephemerides among other trajectory constraints and projected performance parameters for the propulsion systems has delayed structuring the computer programs for the unmanned Earth-Mars round trip mission. Emphasis will be placed during the next period on the application of mathematical methods and development of programs that will permit the early realization of this capability.

An undergraduate student in the Department of Geological Engineering, Mr. Thomas C. Hanks, has undertaken a study of the requirements for subsurface exploration of Mars as a preliminary to determining the mass, volume and power requirements of an automatic drilling and coring machine. In our research such a machine is of interest as a possible payload component for the unmanned Earth-Mars round trip, but the areologic reasons for drilling and taking cores of the Martian crust must first be studied.

APPENDIX A: References

1. Kelley, H. J., Method of Gradients, Optimization Techniques with Applications to Aerospace Systems, edited by G. Leitmann (Academic Press, Inc., New York, 1962), 1st ed., Chap. VI, p. 246.

2. Kenneth, P. and McGill, R., Two-Point Boundary Value Problem Techniques, to appear as a chapter in Advances in Control Systems: Theory and Applications, Vol. III, edited by C. T. Leondes (Academic Press, Inc., New York).

3. Pines, S., Constants of the Motion for Optimal Thrust Trajectories in a Central Force Field, AIAA J., 2, No. 11, 2010-2014 (1964). See also paper with same title, joint AIAA-IMS-SIAM-ONR Symposium on Control and System Optimization, Monterey, California, Jan. 27-29, 1964.

4. Handelsman, M., Optimal Free-Space Fixed-Thrust Trajectories Using Impulsive Trajectories as Starting Iteratives. (In preparation).

5. Lee, G., An Analysis of Two-Impulse Orbital Transfer, AIAA J., 2, No. 10, 1767-1773, (1964).

6. Handelsman, M., Aerospace Systems Optimization, Guggenheim Laboratories Seminar, Department of Aerospace and Mechanical Sciences, Princeton University, September 1964-May 1965, pp. 251-278.

7. Rocklin, S. M., Investigation of Double Minima in the Impulse Function of a Two-Impulse, Fixed Angle Orbital Transfer, MARS Memo No. 20, Department of Aerospace and Mechanical Sciences, Princeton University, 23 August 1965.

8. Hazelrigg, G. A., Fixed-Time, Two-Impulse, Interplanetary Transfers Using a Sphere-of-Influence Concept, unpublished, MARS Memo No. 19, Department of Aerospace and Mechanical Sciences, Princeton University, 8 June 1965.

9. Jordan, J. F., The Application of Lambert's Theorem to the Solution of Interplanetary Transfer Problems, Tech. Rpt. No. 32-521, Jet Propulsion Laboratory, Institute of Technology, Pasadena, California, Feb. 1, 1964.

10. Hazelrigg, G. A., Computation of Planetary Ephemerides, MARS Memo No. 13, Department of Aerospace and Mechanical Sciences, Princeton University, 20 May 1964.

11. McGill, R. and Kenneth, P., Solution of Variational Problems by Means of a Generalized Newton-Raphson Operator, AIAA J., 2, No. 10, 1761-1766 (1964).

12. MacKay, J. S., Rossa, L. G. and Zimmerman, A. V., Optimum Low-Acceleration Trajectories for Earth-Mars Transfer, Proceedings I.A.S. Symposium Vehicle Systems Optimization, Garden City, N. Y., November 28-29, 1961. Also by same authors, Optimum Low-Acceleration Trajectories for Interplanetary Transfers, NASA Technical Note D-1456, January 1963.

APPENDIX C: List of Publications, As of 30 September 1965

1. Constantine, R. W., An Analysis of a Ramjet Propelled Recoverable Launch Vehicle Stage, (Senior Thesis - June 1964), Princeton Aeronautical Engineering Report No. 717a, (Limited Distribution), 3 June 1964.

2. Richardson, W. P., Investigations of a Hybrid Rocket Powered Unmanned Mars Excursion Vehicle, (Senior Thesis - June 1964), Princeton Aeronautical Engineering Report No. 717b, (Limited Distribution), 3 June 1964.

3. Handelsman, M., Hazelrigg, G. A., Hoffman, L. L. and Wallack, P. J., Calculus of Variations Computation of Two Dimensional Heliocentric Orbit Transfers - Volume 1, Overall Presentation, Princeton Aeronautical Engineering Report No. 717c-1, 29 January 1965.

4. Wallack, P. J., Calculus of Variations Computation of Two Dimensional Heliocentric Orbit Transfers - Volume 2, Computer Program, Princeton Aeronautical Engineering Report No. 717c-2, 29 January 1965.

5. Hoffman, L. L., Calculus of Variations Computation of Two Dimensional Heliocentric Orbit Transfers - Volume 3, Tabulated Results for Earth-to-Mars Transfer, Princeton Aeronautical Engineering Report No. 717c-3, 29 January 1965.

6. (Burton, C. D. and Evans, J. A., Nuclear Space Power System Analyses for the Unmanned Mars Round Trip, [Joint MSE Thesis - May 1965], Princeton Aeronautical Engineering Report No. 717d, [Limited Distribution], 20 May 1965).